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OPTIMIZATION AND GUIDANCE OF FLIGHT TRAJECTORIES  
FOR THE NATIONAL AEROSPACE PLANE

by

A. MIELE

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Final Report on NASA Grant No. NAG-1-1029,  
Optimization and Guidance of Flight Trajectories  
for the National Aerospace Plane<sup>1,2</sup>

by

A. Miele<sup>3</sup>

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<sup>1</sup>Period from June 22, 1989 to December 31, 1990.

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<sup>3</sup>Foyt Family Professor of Aerospace Sciences and Mathematical Sciences, Aero-Astronautics Group, Rice University, Houston, Texas.

Abstract. This paper summarizes the research on optimal trajectories for the National Aerospace Plane performed by the Aero-Astronautics Group of Rice University during the period from June 22, 1989 to December 31, 1990. It is assumed that the aerospace plane is controlled via the angle of attack and the power setting. The time history of the controls is optimized simultaneously with the switch times from one powerplant to another and the final time. The intent is to arrive at NASP guidance trajectories exhibiting many of the desirable characteristics of NASP optimal trajectories.

Key Words. Flight mechanics, hypervelocity flight, atmospheric flight, optimal trajectories, aerospace plane, sequential gradient-restoration algorithm.

Notations

$a$  = acceleration,  $\text{ft/sec}^2$ ;  
 $g_e$  = sea-level acceleration of gravity,  $\text{ft/sec}^2$ ;  
 $h$  = altitude,  $\text{ft}$ ;  
 $I_{sp}$  = specific impulse,  $\text{sec}$ ;  
 $m$  = mass,  $\text{lbf sec}^2/\text{ft}$ ;  
 $M$  = Mach number;  
 $q$  = dynamic pressure,  $\text{lbf/ft}^2$ ;  
 $Q$  = heating rate,  $\text{BTU/ft}^2\text{sec}$ ;  
 $S$  = reference surface area,  $\text{ft}^2$ ;  
 $S_e$  = combustor cross-sectional area,  $\text{ft}^2$ ;  
 $t$  = dimensionless time;  
 $T$  = thrust,  $\text{lbf}$ ;  
 $V$  = velocity,  $\text{ft/sec}$ ;  
 $W$  =  $mg_e$  = sea-level weight,  $\text{lbf}$ ;  
 $x$  = distance along the Earth surface,  $\text{ft}$ ;  
 $\alpha$  = angle of attack,  $\text{rad}$ ;  
 $\beta$  = power setting;  
 $\gamma$  = path inclination,  $\text{rad}$ ;  
 $\theta$  = running time,  $\text{sec}$ ;  
 $\tau$  = final time,  $\text{sec}$ .

Subscripts (EM1 + EM2)

0 = beginning of ramjet phase/initial point;  
1 = beginning of scramjet phase;  
2 = end of scramjet phase/final point;  
f = final point.

Subscripts (EM3)

0 = beginning of ramjet phase/initial point;  
1 = beginning of scramjet phase;  
2 = beginning of rocket phase;  
3 = end of rocket phase/final point;  
f = final point.

Acronyms

GHAME = general hypersonic aerodynamics model example;  
NASP = national aerospace plane;  
SGRA = sequential gradient-restoration algorithm;  
SSTO = single-stage-to-orbit;  
TSTO = two-stage-to-orbit.

## 1. Introduction

The National Aerospace Plane (NASP) is a hypervelocity vehicle which must take-off horizontally, achieve orbital speed, and then land horizontally. At this time, its configuration is not precisely known, but it can be assumed that the powerplant includes the combination of four types of engines: turbojet or turbofan engines for flight at subsonic speeds and low supersonic speeds; ramjet engines for flight at high supersonic speeds; scramjet engines for flight at hypersonic speeds; and rocket engines for flight at near-orbital speeds.

In this research, optimal trajectories are studied for a given NASP configuration, the so-called general hypersonic aerodynamics model example (GHAME). The optimization study is done employing the sequential gradient-restoration algorithm for optimal control problems. This algorithm, developed and perfected by the Aero-Astronautics Group of Rice University over the years 1970-85, has proved to be a powerful and reliable tool for solving highly constrained/highly nonlinear problems of optimal control, such as flight in a windshear and aeroassisted orbital transfer. Here, it is applied to the NASP problem.

In a subsequent research, the current optimization study is to be followed by a guidance study. The intent is to develop guidance trajectories, capable of approximating the key properties of the optimal trajectories. This is to be achieved via a feedback control scheme characterized by strong resistance

to external disturbances. See, for instance, the guidance schemes already developed by the Aero-Astronautics Group of Rice University for flight under windshear conditions and aeroassisted orbital transfer.



## 2. Research Results

The research involves 48 optimization problems obtained by combining several performance indexes [P1, P2, P3, P4], constraint combinations [A, B, C, D], and engine models [EM1, EM2, EM3].

The performance indexes being minimized are four:

- (P1) weight of fuel consumed;
- (P2) peak dynamic pressure;
- (P3) peak heating rate;
- (P4) peak tangential acceleration.

The constraint combinations are four:

- (A)  $\gamma_0 = \text{free}$ ,  $q = \text{free}$ ,  $a_T = \text{free}$ ;
- (B)  $\gamma_0 = \text{free}$ ,  $q \leq 1500 \text{ lbf/ft}^2$ ,  $a_T \leq 3g_e$ ;
- (C)  $\gamma_0 = 0.0 \text{ deg}$ ,  $q = \text{free}$ ,  $a_T = \text{free}$ ;
- (D)  $\gamma_0 = 0.0 \text{ deg}$ ,  $q \leq 1500 \text{ lbf/ft}^2$ ,  $a_T \leq 3g_e$ .

The engine models are three:

(EM1) this is a ramjet/scramjet combination in which the scramjet specific impulse tends to a nearly-constant value at large Mach numbers;

(EM2) this is a ramjet/scramjet combination in which the scramjet specific impulse decreases monotonically at large Mach numbers;

(EM3) this is a ramjet/scramjet/rocket combination in which, owing to stagnation temperature limitations, the scramjet operates only at  $M \leq 15$ ; at higher Mach numbers, the scramjet is shut off and the aerospace plane is driven only by the rocket engines.

Note that a peak heating rate bound,  $Q \leq 150 \text{ BTU/ft}^2\text{sec}$ , is not imposed because it can be satisfied or nearly satisfied indirectly if the dynamic pressure bound is satisfied.

The minimization of the above performance indexes is carried out under the assumption that the turbojet phase has been completed. The initial conditions are

$$x_0 = 0 \text{ ft}, \quad (1a)$$

$$h_0 = 42000 \text{ ft} = 12.8 \text{ km}, \quad (1b)$$

$$V_0 = 1936 \text{ ft/sec}, \quad (1c)$$

$$\gamma_0 = \text{free or } \gamma_0 = 0.0 \text{ deg}, \quad (1d)$$

$$W_0 = 290000 \text{ lbf}, \quad (1e)$$

and correspond to  $M_0 = 2$ ,  $q_0 = 1000 \text{ lbf/ft}^2$ . The final conditions are

$$x_f = \text{free}, \quad (2a)$$

$$h_f = 262467 \text{ ft} = 80.0 \text{ km}, \quad (2b)$$

$$V_f = 25792 \text{ ft/sec}, \quad (2c)$$

$$\gamma_f = 0.0 \text{ deg}, \quad (2d)$$

$$W_f = \text{free}, \quad (2e)$$

and correspond to  $M_f = 27.8$ ,  $q_f = 11.9 \text{ lbf/ft}^2$ , and orbital speed.

It is assumed that the NASP reference surface area is  $S = 6000 \text{ ft}^2$  and that the combustor cross-sectional area of both the ramjet and the scramjet is  $S_e = 400 \text{ ft}^2$ . It is further assumed that the NASP is controlled via the angle of attack  $\alpha(t)$  and the power setting  $\beta(t)$ , which are subject to the inequalities

$$-2.0 \leq \alpha \leq 12.0 \text{ deg}, \quad (3a)$$

$$0.0 \leq \beta \leq 1.0. \quad (3b)$$

Finally, it is assumed that the ramjet specific impulse has the maximum value  $I_{sp} \approx 6000$  sec for all engine models; that the scramjet specific impulse has the maximum value  $I_{sp} \approx 2500$  sec for engine model EM1 and  $I_{sp} \approx 3100$  sec for engine models EM2 and EM3; and that the rocket specific impulse has the constant value  $I_{sp} = 444$  sec for engine model EM3. The rocket maximum thrust is  $T = 189200$  lbf for engine model EM3.

For detailed data and results, see Refs. 1-2. A cross section of the results obtained is shown in Tables 1-3, each containing a different group of problems [G1, G2, G3]. In analyzing the results of Tables 1-3, the criteria for judging the engineering usefulness of the solutions are as follows:

(C1) the initial path inclination  $\gamma_0$  should be small to avoid overburdening the turbojet engines during the low-altitude portion of the flight;

(C2) the dynamic pressure  $q$  should be kept below 1500 lbf/ft<sup>2</sup>;

(C3) the heating rate  $Q$  should be maintained below 150 BTU/ft<sup>2</sup>sec;

(C4) the tangential acceleration  $a_T$  should not exceed  $3g_e$ .

Group G1. This group of problems is concerned with the effect of the performance index and includes Problems P1A, P2A, P3A, P4A. Each of the performance indexes P1-P4 is minimized for

constraints of type A and engine model EM1. Clearly, unconstrained solutions are obtained, since the initial path inclination, the dynamic pressure, and the tangential acceleration are free. See Table 1.

Among the solutions of group G1, solutions P1A, P2A, P3A lead to excessive violation of criteria C1 and C4, while solution P4A is unacceptable in the light of criteria C1-C3. A common characteristic of the solutions of group G1 is the steepness of the trajectory at the initial point: values of  $\gamma_0$  ranging from 38.3 deg to 50.0 deg are obtained.

Group G2. This group of problems is concerned with the effect of the constraint combination and includes Problems P1A, P1B, P1C, P1D. The performance index P1 is minimized for constraints of type A, B, C, D and engine model EM1. See Table 2.

Solutions P1A and P1B lead to excessive violation of criterion C1. Unacceptable values of the initial path inclination are obtained, specifically,  $\gamma_0 = 42.0$  deg and  $\gamma_0 = 39.4$  deg; this is not surprising, since  $\gamma_0$  is left free.

Solutions P1C and P1D are computed for  $\gamma_0 = 0.0$  deg; hence, they meet automatically criterion C1. However, solution P1C leads to excessive violation of criteria C2-C4. The only solution consistent with criteria C1-C4 is solution P1D.

If one compares the time histories of the dynamic pressure and the heating rate for solutions P1A and P1D, one sees that they have something in common: a fast initial climb to quickly

decrease the air density, so as to contain both the dynamic pressure and the heating rate. In solution PlA, this is obtained with  $\gamma_0 = 42.0$  deg; in solution PlD, this is obtained with a quick increase of the path inclination from  $\gamma_0 = 0.0$  deg to  $\gamma_0 = 25.0$  deg. While the unconstrained solution PlA exceeds the tangential acceleration bound, the constrained solution PlD satisfies the tangential acceleration bound by reducing the power setting when the  $3g_e$  limit is met. The fuel penalty paid for imposing the additional constraints concerning  $\gamma_0$ ,  $q$ ,  $a_T$  is only 2% as can be seen by comparing solutions PlA and PlD.

Group G3. This group of problems is concerned with the effect of the engine model and includes three solutions of Problem PlD. The performance index Pl is minimized for constraints of type D and engine models EM1, EM2, EM3. See Table 3.

Clearly, all the solutions of Group G3 satisfy criteria C1-C4. In percentage of the aerospace plane weight at the end of the turbojet phase, the minimum fuel weight is 34.3% for engine model EM1, 44.3% for engine model EM2, and 60.7% for engine model EM3. Let us assume that the turbojet portion of the flight consumes a fuel weight equal to 5.0% of the take-off weight. Then, in percentage of the aerospace plane weight at take-off, the minimum fuel weight is 37.6% for engine model EM1, 47.1% for engine model EM2, and 62.7% for engine model EM3.

The fact that engine model EM3 carries a severe fuel weight penalty is clear. Since the scramjet operation is discontinued at

M = 15, the flight portion from M = 15 to M = 27.8 is spent under rocket power. Because the largest amount of energy increase takes place during the rocket phase and because the rocket engine has lower specific impulse, engine model EM3 uses 33% of the fuel weight in the ramjet-scramjet phase and 67% in the rocket phase. Indeed, the consequences of carrying the oxidizer onboard are severe for this type of SSTO vehicle.

These results indicate that the required fuel weight and connectedly the useful payload are dependent heavily on the performance of the scramjet powerplant at hypersonic Mach numbers. If engine model EM2 is closer to reality, then the SSTO mission appears to be feasible. On the other hand, if engine model EM3 is closer to reality, then the SSTO mission appears to be marginal at best.

Of course, improvements are possible in the areas of aerodynamic properties and specific impulse properties via highly-integrated airframe/engine combinations. Under this scenario, the SSTO mission might become feasible. But prudence seems to dictate that a TSTO mission deserves concurrent consideration.

Table 1. Group G1 solutions, effect of the performance index, constraints of Type A, engine model EM1.

Quantity	Problem				Units
	P1A	P2A	P3A	P4A	
$(W_0 - W_f)/W_0$	0.337	0.347	0.357	0.550	-
$\max(q)$	1540	999	1157	3751	lbf/ft <sup>2</sup>
$\max(Q)$	165	161	98	495	BTU/ft <sup>2</sup> sec
$\max(a_T)/g_e$	9.1	5.2	4.0	1.1	-
$\gamma_0$	42.0	50.0	40.4	38.3	deg
$\tau_1$	34	54	48	144	sec
$\tau_2$	409	475	731	704	sec
$\theta_f$	443	529	779	848	sec
$W_f = W_2$ and $\theta_f = \theta_2$ for engine model EM1.					

Table 2. Group G2 solutions, effect of the constraint combination, minimum fuel weight, engine model EM1.

Quantity	Problem				Units
	P1A	P1B	P1C	P1D	
$(W_0 - W_f)/W_0$	0.337	0.340	0.339	0.343	-
$\max(q)$	1540	1112	1765	1500	lbf/ft <sup>2</sup>
$\max(Q)$	165	148	200	153	BTU/ft <sup>2</sup> sec
$\max(a_T)/g_e$	9.1	3.0	13.7	3.0	-
$\gamma_0$	42.0	39.4	0.0	0.0	deg
$\tau_1$	34	55	34	55	sec
$\tau_2$	409	498	335	487	sec
$\theta_f$	443	553	369	542	sec
$W_f = W_2$ and $\theta_f = \theta_2$ for engine model EM1.					



Table 3. Group G3 solutions, effect of the engine model,  
minimum fuel weight, constraints of Type D.

Quantity	Engine model			Units
	EM1	EM2	EM3	
$(W_0 - W_f)/W_0$	0.343	0.443	0.607	-
$\max(q)$	1500	1425	1500	lbf/ft <sup>2</sup>
$\max(Q)$	153	157	110	BTU/ft <sup>2</sup> sec
$\max(a_T)/g_e$	3.0	3.0	3.0	-
$\gamma_0$	0.0	0.0	0.0	deg
$\tau_1$	55	44	57	sec
$\tau_2$	487	472	97	sec
$\tau_3$	-	-	277	sec
$\theta_f$	542	517	431	sec

$W_f = W_2$  and  $\theta_f = \theta_2$  for engine models EM1, EM2.

$W_f = W_3$  and  $\theta_f = \theta_3$  for engine model EM3.

### 3. Abstracts of Publications

- 3.1. MIELE, A. LEE, W. Y., and WU, G. D., Optimal Trajectories for an Aerospace Plane, Part 1: Formulation, Results, and Analysis, Rice University, Aero-Astronautics Report No. 247, 1990.

Abstract. This report is concerned with the optimization of the trajectories of an aerospace plane. This is a hypervelocity vehicle capable of achieving orbital speed, while taking off horizontally. The vehicle is propelled by four types of engines: turbojet engines for flight at subsonic speeds/low supersonic speeds; ramjet engines for flight at moderate supersonic speeds/low hypersonic speeds; scramjet engines for flight at hypersonic speeds; and rocket engines for flight at near-orbital speeds.

A single-stage-to-orbit (SSTO) configuration is considered, and the transition from low supersonic speeds to orbital speeds is studied under the following assumptions: the turbojet portion of the trajectory has been completed; the aerospace plane is controlled via the angle of attack  $\alpha(t)$  and the power setting  $\beta(t)$ ; the aerodynamic model is the generic hypersonic aerodynamics model example (GHAME). Concerning the engine model, three options are considered: (EM1) this is a ramjet/scramjet combination in which the scramjet specific impulse tends to a nearly-constant value at large Mach numbers; (EM2) this is a ramjet/scramjet combination in which the scramjet specific impulse decreases monotonically at large Mach

numbers; (EM3) this is a ramjet/scramjet/rocket combination in which, owing to stagnation temperature limitations, the scramjet operates only at  $M \leq 15$ ; at higher Mach numbers, the scramjet is shut off and the aerospace plane is driven only by the rocket engines.

Under the above assumptions, four optimization problems are solved using the sequential gradient-restoration algorithm for optimal control problems: (P1) minimization of the weight of fuel consumed; (P2) minimization of the peak dynamic pressure; (P3) minimization of the peak heating rate; and (P4) minimization of the peak tangential acceleration. The above optimization studies are carried out for different combinations of constraints, specifically: initial path inclination either free or given ( $\gamma_0 = 0$ ); dynamic pressure either free or bounded ( $q \leq 1500 \text{ lbf/ft}^2$ ); tangential acceleration either free or bounded ( $a_T \leq 3g_e$ ).

The main conclusions are as follows:

(a) For an aerospace plane governed by GHAME + EM1, the SSTO mission requires a weight of fuel consumed equal to 34.3% of the initial weight.

(b) For an aerospace plane governed by GHAME + EM2, the SSTO mission requires a weight of fuel consumed equal to 44.3% of the initial weight.

(c) For an aerospace plane governed by GHAME + EM3, the SSTO mission requires a weight of fuel consumed equal to 60.7% of the initial weight.

(d) If one assumes that engine model EM2 is the one closer to reality, then the SSTO mission appears to be feasible. Obviously, its ability to deliver payloads can be improved via progress in the areas of aerodynamic properties and specific impulse properties.

(e) If one assumes that engine model EM3 is the one closer to reality, then the SSTO mission appears to be marginal, unless substantial progress is achieved in the areas of aerodynamic properties and specific impulse properties. Under this scenario, alternative consideration should be given to studying the feasibility of a two-stage-to-orbit (TSTO) mission.

3.2. MIELE, A., LEE, W. Y., and WU, G. D., Optimal Trajectories for an Aerospace Plane, Part 2: Data, Tables, and Graphs, Rice University, Aero-Astronautics Report No. 248, 1990.

Abstract. This report is a follow-up to Ref. 1 and presents data, tables, and graphs relative to the optimal trajectories for an aerospace plane. A single-stage-to-orbit (SSTO) configuration is considered, and the transition from low supersonic speeds to orbital speeds is studied for a single aerodynamic model (GHAME) and three engine models.

Four optimization problems are solved using the sequential gradient-restoration algorithm for optimal control problems: (P1) minimization of the weight of fuel consumed; (P2) minimization of the peak dynamic pressure; (P3) minimization of the peak heating rate; and (P4) minimization of the peak tangential acceleration. The above optimization studies are carried out for different

combinations of constraints, specifically: initial path inclination either free or given; dynamic pressure either free or bounded; tangential acceleration either free or bounded.

ReferencesReports

1. MIELE, A., LEE, W. Y., and WU, G. D., Optimal Trajectories for an Aerospace Plane, Part 1: Formulation, Results, and Analysis, Rice University, Aero-Astronautics Report No. 247, 1990.
2. MIELE, A., LEE, W. Y., and WU, G. D., Optimal Trajectories for an Aerospace Plane, Part 2: Data, Tables, and Graphs, Rice University, Aero-Astronautics Report No. 248, 1990.